NAG Toolbox for MATLAB

f08aa

1   Purpose

f08aa solves linear least-squares problems of the form

$$\min_x \|b - Ax\|_2 \qquad \text{or} \qquad \min_x \|b - A^{\mathrm{T}}x\|_2,$$

where A is an m by n matrix of full rank, using a QR or LQ factorization of A.

2   Syntax

```
[a, b, info] = f08aa(trans, a, b, 'm', m, 'n', n, 'nrhs_p', nrhs_p)
```

3   Description

The following options are provided:

1. If **trans** = 'N' and $m \geq n$: find the least-squares solution of an overdetermined system, i.e., solve the least-squares problem

    $$\min_x \|b - Ax\|_2.$$

2. If **trans** = 'N' and $m < n$: find the minimum norm solution of an underdetermined system $Ax = b$.

3. If **trans** = 'T' and $m \geq n$: find the minimum norm solution of an undetermined system $A^{\mathrm{T}}x = b$.

4. If **trans** = 'T' and $m < n$: find the least-squares solution of an overdetermined system, i.e., solve the least-squares problem

    $$\min_x \|b - A^{\mathrm{T}}x\|_2.$$

Several right-hand side vectors b and solution vectors x can be handled in a single call; they are stored as the columns of the m by r right-hand side matrix B and the n by r solution matrix X.

4   References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D 1999 *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia URL: http://www.netlib.org/lapack/lug

Golub G H and Van Loan C F 1996 *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5   Parameters

5.1   Compulsory Input Parameters

1:    **trans – string**

   If **trans** = 'N', the linear system involves A.

   If **trans** = 'T', the linear system involves A^{T}.

   Constraint: **trans** = 'N' or 'T'.

2:    **a(lda,∗) – double array**

   The first dimension of the array **a** must be at least $\max(1, \mathbf{m})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The m by n matrix A.

3:      **b(ldb,∗) − double array**

The first dimension of the array **b** must be at least $\max(1, \mathbf{m}, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{nrhs\_p})$

The matrix B of right-hand side vectors, stored in columns; **b** is m by r if **trans** = 'N', or n by r if **trans** = 'T'.

5.2   Optional Input Parameters

1:      **m − int32 scalar**

Default: The first dimension of the array **a**.

m, the number of rows of the matrix A.

Constraint: $\mathbf{m} \geq 0$.

2:      **n − int32 scalar**

Default: The second dimension of the array **a**.

n, the number of columns of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

3:      **nrhs_p − int32 scalar**

Default: The second dimension of the array **b**.

r, the number of right-hand sides, i.e., the number of columns of the matrices B and X.

Constraint: $\mathbf{nrhs\_p} \geq 0$.

5.3   Input Parameters Omitted from the MATLAB Interface

lda, ldb, work, lwork

5.4   Output Parameters

1:      **a(lda,∗) − double array**

The first dimension of the array **a** must be at least $\max(1, \mathbf{m})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

If $\mathbf{m} \geq \mathbf{n}$, **a** contains details of its QR factorization as returned by f08ae.

If $\mathbf{m} < \mathbf{n}$, **a** contains details of its LQ factorization as returned by f08ah.

2:      **b(ldb,∗) − double array**

The first dimension of the array **b** must be at least $\max(1, \mathbf{m}, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{nrhs\_p})$

b contains the solution vectors, x, stored in columns:

> if **trans** = 'N' and $m \geq n$, or **trans** = 'T' and $m < n$, elements 1 to $\min(m, n)$ in each column of **b** contain the least squares solution vectors; the residual sum of squares for the solution is

given by the sum of squares of the modulus of elements $(\min(m, n) + 1)$ to $\max(m, n)$ in that column;

otherwise, elements 1 to $\max(m, n)$ in each column of **b** contain the minimum norm solution vectors.

3:    **info – int32 scalar**

info $= 0$ unless the function detects an error (see Section 6).

6   Error Indicators and Warnings

Errors or warnings detected by the function:

info $= -i$

If **info** $= -i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **trans**, 2: **m**, 3: **n**, 4: **nrhs_p**, 5: **a**, 6: **lda**, 7: **b**, 8: **ldb**, 9: **work**, 10: **lwork**, 11: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

info > 0

If **info** $= i$, diagonal element i of the triangular factor of A is zero, so that A does not have full rank; the least squares solution could not be computed.

7   Accuracy

See Section 4.5 of Anderson *et al.* 1999 for details of error bounds.

8   Further Comments

The total number of floating-point operations required to factorize A is approximately $\frac{2}{3}n^2(3m - n)$ if $m \geq n$ and $\frac{2}{3}m^2(3n - m)$ otherwise. Following the factorization the solution for a single vector x requires $O(\min(m^2, n^2))$ operations.

The complex analogue of this function is f08an.

9   Example

```
trans = 'No transpose';
a = [-0.57, -1.28, -0.39, 0.25;
     -1.93, 1.08, -0.31, -2.14;
     2.3, 0.24, 0.4, -0.35;
     -1.93, 0.64, -0.66, 0.08;
     0.15, 0.3, 0.15, -2.13;
     -0.02, 1.03, -1.43, 0.5];
b = [-2.67;
     -0.55;
     3.34;
     -0.77;
     0.48;
     4.1];
[aOut, bOut, info] = f08aa(trans, a, b)

aOut =
    3.6177   -0.5566    0.8474    0.7460
    0.4609   -2.0281    0.5514    1.1700
```

```
     -0.5492    -0.0457     1.3745    -1.4105
      0.4609     0.2828     0.0044    -2.3755
     -0.0358     0.0796    -0.0773    -0.5214
      0.0048     0.3003     0.8017     0.2558
bOut =
      1.5339
      1.8707
     -1.5241
      0.0392
     -0.0085
      0.0204
info =
           0
```